## NOTICIAS de TODOS

 TODOS Newsletter: Mathematics for ALL
## Winter 2017, Volume 12, Number 2

## See TODOS on line

www.todos-math.org


## @todosmath

## Taking a Stand for Humanity

TODOS: Mathematics for ALL is an organization that seeks to create a more just, humanizing, and equitable mathematics education experience for all. Regardless of your political views, we cannot let our differences overshadow our humanity toward each other. We recognize that the current political climate may affect how we move forward as a people that value democracy and justice for all. We must find strength and resolve to reach out to people hurting, scared, and uncertain of their futures. We must find ways to support educators to hold space for listening, emotions, and deeper understanding. We have much work to do.

We reiterate here our TODOS mission and goals. In the present political climate, we interpret these as including the following:

- Respecting and incorporating into our mathematics programs, the role language and culture play in teaching and learning mathematics.
- Supporting teachers who need help navigating the political and emotional situations occurring daily in their classrooms.
- Generating and disseminating knowledge that supports our mission of advocacy for all students.
- Informing the public and influencing educational policies that protect our students and enhance the educational experiences of all of our students.
- Informing families about the opportunities available to their children and working continuously and ardently to enable these children to become mathematically proficient.
As mathematics educators, we will continue to stand with our students and their families, advocate for them, and affirm their futures.

Diane Kinch<br>President, TODOS: Mathematics for $\boldsymbol{A} L \boldsymbol{L}$

From the President<br>By Diane Kinch<br>dokinch@gmail.com

Our successful 2016 TODOS Conference, Ensuring Equity and Excellence in
 Mathematics for All continued to address the mission and goals of TODOS. I would like to share with you how our attendees reflected on their experiences regarding these matters.

## Incorporating Language and Culture in Teaching and Learning Mathematics <br> Culture



- "I really know very little about indigenous cultures. This session gave me new insights about the lives of my Native students."
- "There was a great discussion about including culture and funds of knowledge to bring your students into the lessons. I liked the idea of integrating backgrounds and folk into what is being learned or thinking though lessons with that lens."
- "I think incorporating more reading examples from different cultures will help my students develop their own identities and feel a sense of wholeness."
- "We have to embrace the different ways all cultures teach mathematics."

Kinch, continued on page 4

## Winter 2017, Volume 12, Number 2

In this issue:

- Taking a Stand for Humanity, p. 1
- News from the President, p. 1
- Solving the Re-contextualized Problem, p. 2
- A Call for Action, p. 4
- TODOS Live!, p. 4
- Meeting Our TODOS Goal for Families, p. 7

TODOS Mission Statement
The mission of TODOS: Mathematics for ALL is to advocate for equity and high quality mathematics education for all students -in particular, Latina/o students.

## SOLVING the Re-contextualized Problem: Teaching Open-Ended Math Problems within a Context of a Social Justice Issues <br> By Cecilia Lopez, Mathematics Teacher Pioneer High School, WJUSD Woodland, CA cecilia.lopez@wjusd.org. <br> Edited by Susana Davidenko

The article Teaching Open-Ended Math Problems within a Context of a Social Justice Issue appeared in the Spring 2016 issue of Noticias:
https://toma.memberclicks.net/assets/documents/noticias/ noticiasv12n1spring2016.pdf

In that article, Cecilia Lopez highlighted some advantages of contextualizing mathematics problems in a way that they relate to students' background knowledge, experiences, and/or interests. Teachers wanting to bringing the conversation about social justice matters into the mathematics classroom could use this approach while teaching the math content of their courses.
Please read or revisit the original article for more suggestions about supporting students to express their views and ideas.
As an example of De Freitas' (2008) suggestion of modifying the context of a (known or published) problem, Lopez's article in Noticias included the following problem and modification.

## Original Problem: Cost of a Camping Trip

A group of friends rented a place for a camping trip. A week before the trip 4 friends had to withdraw from the trip. The remaining participants will then have to pay $\$ 5$ more per person. Then two days before the trip 2 more friends realized they cannot make the trip and let the group know that they will not pay. As a result, the remaining friends have to pay $\$ 3$ extra each.

How many friends went to the trip?
How much money did they pay?
From: http://www.rekenbeter.nl/EerdereSommen.aspx

## Problem within a Modified Context: Lay Offs \& Increasing Working Hours

A company hired temporary workers to complete a building project and assigned a given number of hours to each worker. Two days before the day the project was expected to start, the manager fired 4 workers and assigned 5 more hours to each of the remaining workers. Then, the night before the job was to begin, the manager decided to fire 2 more workers and assign three more hours to the remaining ones.

How many workers worked on the project? How many hours did each of them work? Modified Problem Written by Cecilia Lopez

## Building Background Knowledge and Vocabulary

Note: the modified problem might be more relevant to the students than the original one. However, the language complexity is still a challenge for English learners. Teachers should simplify the wording to facilitate understanding. For example, they can write the problem in short statements and elaborate on each statement before moving to the next. For example: "A company hired temporary workers to complete a job. Each worker got the same number of hours. Before starting the job, 4 workers were fired" and so on.

The teacher should read the problem out loud one or two times and clarify terms to help students build background knowledge. Students must understand the meaning of hiring/firing; full time/ part time and temporary workers; and wage/hour, earnings, hours/week, etc. Immigrant students and ELLs might not be familiar with workers' responsibilities and rights; clarifications and proper information will contribute to students' understanding of expectations for fair workplace treatment.

Developing the previous vocabulary before the lesson would also be of great help to ELLs. This could be done in collaboration with the ESL teacher or other school personnel that support ELLs.

Following are examples of different approaches that students might use to understand the problem, discover what variables are involved and how they are related and, finally, how to solve the problem.

## Role-Playing and/or Using Manipulatives to Recreate the Context of the Problem

Students who have difficulties interpreting the problem might benefit by representing the situation using themselves as workers and counters as hours (role playing) or using paper plates and counters to represent workers and hours per worker.
Since the problem does not give the number of workers and hours (these are the unknowns of the problem), students can first represent a situation in which the number of workers and number of hours are given. Then, they can follow steps similar to the ones in the problem and see how the number of workers and hours change, while the total number of hours is fixed.

Example: 5 workers are hired and each worker gets $\mathbf{1 0}$ hours of work. Before starting the job, one worker is fired and the boss re-assigns that worker's hours to remaining workers. Finally, still before starting the job, two more workers are fired and their hours distributed to the remaining workers.

## How many workers will complete the job? How many hours will each work?

In the following diagrams, a large circle represents a worker and a small circle inside the large circle represents an hour.

FIRST situation: 5 workers get 10 hours each.


[^0]
## Lopez, continued

SECOND situation: 1 worker is fired and the boss equally distributes his/her 10 hours to the remaining 4 workers.


## SECOND situation: 4 workers get 12.5 hours each.

 Total hours $4 \times 12.5=50$THIRD, and final, situation: 2 more workers are fired and the boss re-assigns their hours to the remaining 2 workers. These workers had 12.5 each, so 25 are redistributed.

12.5 more hours for each worker will make $12.5+12.5=25$ hours for each worker

THIRD, and final, situation: 2 workers get 25 hours each. Total hours $2 \times 25=50$

After using these concrete or visual representations, students might be able to think more abstractly. Instead of 5 workers, they can consider "W" as the number of workers which can take on any value (whole numbers). When there are some workers fired, students might understand that, since W is not known, the number of remaining workers can be expressed as W - (number of workers fired); if 1 worker was fired, then (W-1). Similarly, expressions with "H" can represent changes in the number of hours.

The next steps will be to create and solve a system of equations. This is detailed on page 5 .

## Rectangular Representation to Relate the Variables

Another visual representation which some students might prefer or the teacher could suggest, is one where the variables in the problem (\# of workers and \# of hours per worker) can be represented as the dimensions of a rectangle. The total number of hours will be represented by the area (which is unknown but fixed.)

FIRST, original situation: An unknown number of workers (W) and each worker got the same (unknown) number of hours $(\mathbf{H})$ to complete a job.
Total hours, $\mathrm{T}=$ number of workers times number of hours per worker. That is, $\mathbf{T}=\mathbf{W} \times \mathbf{H}$ hours


Second situation: 4 workers were fired; remaining workers get 5 more hours each.


Third, and final, situation: 2 more workers were fired; remaining workers get 3 more hours each.


Once students have arrived at this visualization and wrote the expressions and equations, they might be ready for the algebraic solution shown on page 5 .

Lopez, continued on page 4

## Lopez, continued

## Trial and Error Strategy

Some students might want to use a trial and error strategy. Although the strategy would not give much insight to students in this particular problem, students can still make sense of the variables and determine the appropriate expressions for the variables, their changes and relationships.
The table below shows four different attempts to find the solution. A solution to the problem will show three numbers in the $\mathrm{W} x \mathrm{H}$ column that are identical.

| w | H | WxH | $w$ | H | $\boldsymbol{W} \times \mathrm{H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10 | 100 | 15 | 10 | 150 |
| $\begin{gathered} w-4 \\ =6 \end{gathered}$ | $\begin{gathered} H+5= \\ 15 \end{gathered}$ | 90 | $\begin{gathered} W-4 \\ =11 \end{gathered}$ | $\begin{gathered} H+5= \\ 15 \end{gathered}$ | 165 |
| $\begin{gathered} W-6 \\ =4 \end{gathered}$ | $\begin{gathered} \mathrm{H}+8= \\ 18 \end{gathered}$ | 72 | $\begin{gathered} W-6 \\ =9 \end{gathered}$ | $\begin{gathered} H+8= \\ 18 \end{gathered}$ | 162 |
| $\begin{gathered} 10 \\ W-4 \\ =6 \end{gathered}$ | $\begin{gathered} H+5= \\ 17 \end{gathered}$ | 120 102 | 36 $\begin{gathered} W-4 \\ =32 \end{gathered}$ | $\begin{gathered} 40 \\ H+5= \\ 46 \end{gathered}$ | $1,440$ |
| $\begin{gathered} W-6 \\ =4 \end{gathered}$ | $\begin{gathered} H+8= \\ 20 \end{gathered}$ | 80 | $\begin{gathered} W-6 \\ =30 \end{gathered}$ | $\begin{gathered} H+8= \\ 48 \end{gathered}$ | $1,440$ |

Final solution: 30 workers work 48 hours each

Same value for the total hours.
Lopez, continued on page 5

> A Call for Collective Action is a collaborative project of eight mathematics education organizations (AMTE, BBA, CMC-South, NCTM, NCSM, NASGEm, TODOS, and WME ) and the journal JUME.

> The purpose of this project is to have "a year dedicated to building our collective knowledge and understanding of topics and issues related to Equity and Social Justice in Mathematics Education."

Please, join other TODOS members in this process. You will read thought provoking documents that are posted each month and then you can share your perspectives and views with others.

Follow the directions to participate in this common undertaking at: http://www,todos-math.org/a-call-for-collective-action


TODOS Live! Free Webinars for TODOS Members

Season Six has already presented very interesting Webinars and more are coming up!

Keep informed about this opportunity for members at http://www.todos-math.org/todoslive , read the recorded sessions from Season Six and access other archive files.

## Kinch, continued

## Language

- "Treat everyday and home languages as resources, not obstacles."
- "Scaffold the language; keep the math rigor."
- "How am I talking to English learners in class? Am I making assumptions based on their language?"
- "We cannot justify dumbing down the math because our students don't have the language. They have the cognitive ability, they just need the language."
- "I will try to be responsive to the language and culture of the students that I teach. I will try to find ways to collect data that will be telling more about what students can do and inform my teaching practice."
- "I certainly will inform my students that everyone will treat each other with respect, no matter the color of their skin or the language they speak or their cultural background."
- "We are all second language learners when it comes to math."


## Implementing an Equitable, Rigorous, and Coherent Mathematics Program

- "Rigor does not equal hard! Rigor means more meaningful, makes you think, etc. It's actually easy to create more rigorous tasks; you just have to think it through before hand."
- "Rigorous math problems should not be watered down just because the students are learning English. I will make sure to scaffold the language for my English learners and keep the math rigor of the problems."
"Develop a common definition of rigor."
- "Introduce rigorous problem and then incorporate academic English vocabulary."
- "I have been pushed to solve rigorous problems during this conference and it motivates me to challenge my students to increase achievement."
- "I will try to provide rigorous, accessible math instruction for all students."


## Informing Parents, Families and Communities

- "Incorporate more strategies to involve parents as resources."
- "I got ideas about how to engage parents and demonstrate that math outside the classroom is valued."
- "It has given me ideas to bring back to my school. Like the short parent courses. I think parents would like to be involved and learn about what their children are learning."
- "Creating tasks that incorporate math, community issues, and critical thinking will provide students with the opportunity to be passionate about investigating and solving real relevant issues as well as find their confidence in math and power to address those issues."
- "ALL children can learn high level math. Family is still the most important factor in children's success."
- "Parents have strong beliefs about teaching and learning and we need to bring them into the conversation."
- "Use of math in many areas, incorporate funds of knowledge, and math in the homes and family jobs."


## Lopez, continued

## Moving to a More Abstract and General Representation

FIRST situation: An unknown number of workers, where each worker was assigned the same (unknown) number of hours to complete a job.
Naming the variables:
$\mathbf{W}=$ initial, unknown, number of workers
$\mathbf{w}=$ small w represent individual workers
$\mathbf{H}=$ initial, unknown, number of hrs. per worker $\mathbf{T}=$ total number of hours
Workers: w w w w .... w w w w w
Their hrs.: H H H H .... H H H H H H
Total hours needed: $\mathrm{T}=$ Number of workers times Number of hours per worker. Thus:

FIRST situation: T $=\mathbf{W} \times \mathrm{H}$ hours (Equation 1)
SECOND situation: 4 workers were fired; remaining workers were assigned 5 more hours each.
$\mathbf{W}-\mathbf{4}=$ number of remaining workers
$\mathbf{H}+\mathbf{5}=$ new number of hours assigned to each of the $\mathrm{W}-4$ remaining workers


Total hours needed are still the same.
SECOND situation: $T=(W-4) \times(H+5) \quad($ Equation 2)

THIRD (and final) Situation: 2 more workers were fired; remaining workers were assigned 3 more hours each.
$\mathbf{W}-\mathbf{6}=(\mathbf{W}-4)-2$ number of remaining workers
$\mathbf{H}+\mathbf{8}=(\mathbf{H}+\mathbf{5})+\mathbf{3}$ new number of hours assigned to each of the $\mathrm{W}-6$ remaining workers.


Total hours needed are still the same.
THIRD (and final) situation: $\mathbf{T}=(\mathbf{W}-6) \mathbf{x}(\mathbf{H}+8) \quad$ (Eq. 3)
The next steps to solve the problem involve interpreting equations (1), (2), and (3) as information that relates the variables $\mathrm{W}, \mathrm{H}$, and T . Then students will continue manipulating the equations algebraically to find a solution for $\mathrm{W}-6$ and $\mathrm{H}+$ 8 , the final number of workers and their working hours.

NOTE: The substitution of $\mathbf{T}$ by $\mathbf{W} \mathbf{x H}$ is the critical point that turns the relation between W and H into two equations with two unknowns.

The total number of hours, $\mathbf{T}$, needed to complete the job is fixed throughout this process while the number of workers and the number of hours per worker have changed. So we have the three equations,

$$
\begin{align*}
& T=W \times H  \tag{Eq.1}\\
& T=(W-4) \times(H+5)  \tag{Eq.2}\\
& T=(W-6) \times(H+8) \tag{Eq.3}
\end{align*}
$$

Substituting Tby $\boldsymbol{W} \boldsymbol{x} \boldsymbol{H}$ in Eq. 2 and Eq. 3, we can derive the following system of TWO equations with TWO unknowns:
$\begin{cases}\mathbf{W} \times \mathbf{H}=(\mathbf{W}-4) \times(\mathbf{H}+\mathbf{5}) & \text { (Eq. 4) } \\ \mathbf{W} \times \mathbf{H}=(\mathbf{W}-\mathbf{6}) \times(\mathbf{H}+\mathbf{8}) & \text { (Eq. 5) }\end{cases}$
Equations 4 and 5 can be worked out as:
$\mathbf{W} \times \mathbf{H}=(\mathrm{W}-4) \times(\mathrm{H}+5)=\mathbf{W} \times \mathbf{H}+\mathbf{5 W}-\mathbf{4 H}-\mathbf{2 0}$
$\mathbf{W} \times \mathbf{H}=(\mathrm{W}-6) \times(\mathrm{H}+8)=\mathbf{W} \times \mathbf{H}+\mathbf{8 W}-\mathbf{6 H}-\mathbf{4 8}$
We can subtract $\mathrm{W} \times \mathrm{H}$ from each side of each of the equations to obtain the following system:
$\left\{\begin{array}{l}5 W-4 H-20=0 \\ 8 W-6 H-48=0\end{array}\right.$

To solve this system we can multiply (Eq. 6) by 8 and (Eq. 7) by 5 , and then subtract one equation from the other to obtain ONE equation with ONE variable to solve:
(8) $(5 \mathrm{~W}-4 \mathrm{H}-20)=\mathbf{4 0 W}-\mathbf{3 2 H}-\mathbf{1 6 0}=\mathbf{0}($ note, $8 \times 0)$
$(5)(8 \mathrm{~W}-6 \mathrm{H}-48)=\mathbf{4 0 W}-\mathbf{3 0 H}-\mathbf{2 4 0}=\mathbf{0}$

The system

$$
\left\{\begin{array}{l}
\mathbf{4 0 W}-\mathbf{3 2 H}-\mathbf{1 6 0}=\mathbf{0} \text { (Eq. 8) } \\
\mathbf{4 0 W}-\mathbf{3 0 H}-\mathbf{2 4 0}=\mathbf{0} \text { (Eq. } 9)
\end{array}\right.
$$

Subtracting Eq. 8 from Eq. gives us $2 \mathrm{H}-80=0$.
This is now ONE equation with ONE unknown, which we can solve as: $2 \mathrm{H}-80=0 \rightarrow 2 \mathrm{H}=80 \rightarrow \mathbf{H}=\mathbf{4 0}$
That is, the original number of hours per worker was $\mathrm{H}=\mathbf{4 0}$.
Using $\mathrm{H}=40$ in Eq. 8, we obtain:
$40 \mathrm{~W}-32 \times 40-160=40 \mathrm{~W}-1440 \rightarrow 40 \mathrm{~W}=1440 / 40 \rightarrow$ W = $\mathbf{3 6}$ workers

Original: $\mathbf{3 6}$ workers, $\mathbf{4 0}$ hours each. Final answer: $\mathbf{3 0}$ workers, 48 hours each.

## Possible Outline of a Lesson Based on this Problem

## Mathematics Standards

CCSS Mathematics Content Standards are related to expressions, equations and solving simultaneous equations, CCSS: 6.EE-3; 7.EE-4; 8.EE-8; Algebra I-A-CED; A-REI. Mathematical Practices (MP) 1 to 5 could be supported by encouraging students to work together, create representations, problem solve, and share and justify their solutions. States that do not use CCSS would have similar standards.

Lopez, continued on page 6

## Lopez, continued

## Introduction and Lesson CORE

- Access students' previous knowledge and engage with the context of hourly work.
- Read problem aloud and provide needed clarification.
- Allow students a few minutes to think about the problem individually and then work in groups of 2 or 3 .
- Encourage students to represent the problem in any way that helps them understand the scenario, such as role playing, using manipulatives, making diagrams, tables, or illustrations.
- Check in with groups, pose questions, and encourage students to verbalize their reasoning.


## Closing the Lesson

Summarize and reflect on mathematics content and problem context.

- Ask groups to explain their representations and their problem solving process, highlighting what they found challenging.
- Acknowledge each type of representation and approach to a solution, clarifying when needed. This not only empowers all students by validating their work but it also, it provides students with insights into the mathematics discussed in the lesson (MP3). For example, using manipulatives makes it clear that the hours from the fired workers are reassigned to the remaining workers. The rectangular representation shows how a decrease in the number of workers leads to an increase in the number of hours for the remaining workers. Even if creating and working on an algebraic solution might be the objective of the lesson, the process to arrive to the solution is most valuable.
- As groups share their work, highlight all the mathematical ideas that appear such as variables, unknowns, constants, relationships among variables, equations and equivalent equations, linear relationships/functions, systems of two linear equation, and methods to solve the equations.


## Extensions

Follow up with a related or similar problem. For example,

> A small factory has 30 employees. Each employee works 8 hours a day, 5 days per week. The owner needs to reduce the payroll expenses by $20 \%$. How could he do that? What would be a good/ fair arrangement for the workers?

Note: this problem does not lead to a system of linear equations but it focuses on determining the variables and how they are related. These are key understandings for developing knowledge of expressions and equations.
The general situation is that a (simplified) payroll budget $\mathbf{B}$ is:

$$
\mathbf{B}=\mathbf{W} \times \mathbf{H} \times \mathbf{P}
$$

where $\mathbf{W}=\#$ Workers; $\mathbf{H}=\# \mathrm{H} /$ week and $\mathbf{P}=$ pay per hr. Thus, to reduce $20 \%$ the budget B we can reduce $20 \%$ of $\mathrm{W}, \mathrm{H}$ or P (or a combination).

- Reduce the hourly wage by $20 \%$ (e.g., if he pays $\$ 15$ per hour, he will begin to pay $\$ 12$, as $12=0.8 \times 15$, a $20 \%$ less)
- Cut $20 \%$ of the hours per week for each employee; that is, instead of 40 hours per week ( $5 \times 8=40$ ), the would work 32 hours per week ( $32=0.8 \times 40,20 \%$ less)
- Lay-off $20 \%$ of the employees W will turn into .8 W
- Other options?
- Students might do the calculations and verify they arrive at the same reduction of expenses.
Note: When discussing the options, the teacher should (or a student might) notice that reducing the payroll expenses might imply a reduction in production. Reducing the hourly wage might not affect production but then, there might be the need for a negotiation between employers and workers to find a solution.


## Focusing on the Context

- Revisit the initial conversation and ideas related to jobs and types of employment. Encourage students to pose questions regarding workers' rights, hiring /firing processes, minimum wages, equal pay for women, and other issues they might suggest.
- Assign homework so that students spend a few days exploring newspapers or online news or websites related to issues such as minimum wage, furlough, workers' rights, and equal pay for women. These issues can be also related to the presidential candidates' proposals during the recent election period, which makes them still more relevant.
- Note: Teachers should be aware that conversations about earnings and jobs might be delicate for some students, in particular those whose parents are unemployed. Teachers might choose not to talk about these issues or would discuss them in a less personal manner. Thus, the conversation will not highlight students' differences or families' difficulties. It is a priority to respect students' privacy; some students might not want to share personal experiences.
Comments? Questions? Please, email Noticias' editor Susana.Davidenko@cortland.edu


## Kinch, continued

## Developing and Supporting Educational Leaders

- "I can be a leader in my school and district and stand up for all my students and teachers empowered to do more."
- "I want to be a leader for change. All of the students at my school deserve more!"
- "I will be more aware of my role as a leader in the classroom.
- "I will make an effort to identify students that are being overlooked, and I will do my best to ensure that my students know when they are doing well and that I am proud of them."
- "My identity is one of an advocate and leader."


## Mathematics Education Through the Lens of Social Justice

- "There is a need to examine how math can be used to bring about social justice in the classroom."
- "Whether I bring social justice into my teaching or not is a choice and a statement one way or another. What statement do I want to make?"

Kinch, continued on page 7

## Meeting Our TODOS Goal for Families

 By Don Balka, Past President of TODOSThe fifth goal of TODOS: Mathematics for ALL is "To inform families about educational policies and learning strategies that will
 enable their children to become mathematically proficient." I recently spoke at a PTA meeting for a local elementary school where the focus for the event was on mathematics.

A great number of Latino parents and families attended. With very little persuasion, they became involved. Although they attend regularly, they do not participate orally. Activities on a page of random digits engaged many of the Latino parents in attendance. The parents quickly began finding possible number sentences, and were eager to share their success in Spanish with family and friends, and in English with the group. What follows are a few of the activities that were used. They are based on a set of random numbers.

In 1955, the Rand Corporation published a book entitled $A$ Million Random Digits with 100,000 Normal Deviates. It was reissued in 2001 and is now available to download free at http://www.rand.org/pubs/monograph_reports/MR1418.html.

With so many digits, opportunities abound to create activity sheets for parents, students, and teachers to use for number sense concepts.

This is a portion of one page in the book that was used with PTA participants.

$$
\begin{aligned}
& 5 \text { I } 44642756 \\
& 68646 \\
& 8 \\
& 8
\end{aligned} 87029283
$$

The activities that follow are appropriate for students in Grades 1 through 5 , and can be easily modified for other concepts. For parents with children in Grades $\mathrm{K}-5$, the Common Core State Standards covered by the activities are,
Counting and Cardinality (K), Operations and Algebraic Thinking (K-3, 5), Number Operations in Base Ten (K-4).

The first activity involves creating correct number sentences.

## Directions:

1. Select any row and any digit in the row to begin.
2. You can use up to 10 digit (entire row) but the digits must be used in the order in which they appear.
3. Insert operations $(+,-, x, \div)$ between digits and an equal sign $(=)$ to create a correct number sentence.
4. Two or more digits can be combined to create a multi-digit number (e.g., Line 7: 77 or 774).
5. A radical sign can be used to denote square root (e.g., Line 1: $\sqrt{4}=2$ ).
6. Use the symbol ${ }^{\wedge}$ to denote an exponent (e.g., Line 2: $6^{\wedge} 1=6$ ).
7. The number of digits used in a number sentence could be 3 to 10 (e.g., Line 5: $0=4 \times 01 \times 2 \times 8 \times 6 \times 0 \times 7 \times 4$ uses 10 digits).
8. Columns can also be used to create vertical number sentences (e.g., Column 4: $3+1=4$; Column 5: $6-6=0$ ).

Initial results were simple basic facts, a necessary starting point for ALL families engaging their children with the activity. Here are a variety of possible results:

Line 1: $5=1+4$ or $5-1=4 ; 6-4=2$; Line 2: $6+2=8$
Line 3: $7-3=4$; Line 4: $9-5=4 \quad$ Line 5: $4^{\wedge} 0=1$
Line 6: $4=8-4 \quad$ Line 7: $9+24=33$

The second activity involves using more than two numbers to create a correct number sentence, and possibly inserting parentheses. Below are a few results:
Line 1: $5-1+4+4=6+4+2 ; 4=46-42$
Line 2: $64-61=6 \div 2$ Line 3: $85=87+0-2$
Line 4: $9 \times 5=43+2 \quad$ Line 6: $4+8 \div 4=4+0+2$
Line 7: $(4+5)^{\wedge} 1=9$
For families with children in the early primary grades, finding all specified digits is an engaging task (e.g., find all the 7s). Posing questions on finding "hidden" numbers is also important as students learn about place value (e.g., Can you find the number 86 ? (¿Puede encontrar el número 86?) Can you find the numbers from 1 to 20 ? If not, what numbers are missing? (¿Puede encontrar los números del 1 al 20? Si no es así, ¿qué números faltan?)
Selecting one or more numbers: Cutting a slot in a piece of colored paper to show a multi-digit number provides additional activities to engage their children. By moving the paper over the random digits, children can read the number shown (lea el número.), can say if the number is even or is odd (¿El número es par or impar?), and in later grades say that the number is prime or is composite (El número 97 es primo.)

Cutting two slots in the colored paper to show two single digits Allows parents to have children practice on basic addition, subtraction, and multiplication facts
(6 $+9=$ ? $; 9-6=? ; 6 \times 9=$ ? $)$


Balka, continued on page 8

## Kinch, continued

- "It is possible to teach social justice within the math classroom."
- "The positive idea of social justice."
- "It is a good challenge to give a social justice project."
- "Am I holding my students back? How can I promote social justice in the classroom?"
- "I feel more of a responsibility to blend social justice within my math class rather than keep it in lunch conversation, which students tend to want to talk to me about when processing material they learn in social studies."
- "I learned the aspects of power in classroom, culture, curriculum, job and anywhere you go or live. That you just need to learn how to deal with it and use properly."
- "Even though I work hard to promote social justice I am still part of the dominant culture and I need to remember this."

[^1]Balka, continued
Cutting two slots in the colored paper to show two multi-digit numbers provides opportunities for parents to focus on comparison of numbers (e.g., Which number is greater? ¿Cuál de los dos números es más grande?)


Finally, for intermediate grades, cut a longer slot to reveal a multi-digit number such as the one shown. Parents can have their children read the number: Forty-three thousand two hundred ninety-seven.

When children are beginning to focus on the values of various places in a number, parents can ask questions such as What digit is in the thousands place? (¿Qué dígito está en el lugar de los miles?)

NOTICIAS de TODOS - Winter 2017 Vol. 12, No. 2
Editors: Gil Cuevas, Susana Davidenko (Editor), Mary Alice Hatchett, Virginia Nelson and Christine Uliassi.
Contributors: Don Balka, Susana Davidenko, Diane Kinch, Cecilia Lopez, Virginia Nelson, and Christine Uliassi.
Please send items to be published to: Susana.Davidenko@cortland.edu.

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There are many additional activities for other topics in the elementary grades. By cutting a long vertical slot showing two digits covering both sides of the slot with transparent tape, and drawing a horizontal line across the middle, fractions can be formed. Like earlier activities described, children can name the fraction shown. With two slots, they can compare fractions.
In working with parents, a workshop leader or teacher can adjust directions so that all parents can be effective as they learn to use the activities with their children. Understanding directions such as using basic operations $(+,-, x, \div)$ and an equal sign to create a number sentence or being able to compare two numbers (greater than/less than/equal to) empowers parents. The activities presented are simple, yet powerful, and provide opportunities for parents to engage in mathematical conversations at home.

## References

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## Winter 2017, Volume 12, Number 2

In this issue:

- Taking a Stand for Humanity, p. 1
- News from the President, p. 1
- Solving the Re-contextualized Problem, p. 2
- A Call for Action, p. 4
- TODOS Live!, p. 4
- Meeting Our TODOS Goal for Families, p. 7


## PO Box 25482

Tempe AZ 85285-5482


[^0]:    FIRST situation: $\mathbf{5}$ workers get $\mathbf{1 0}$ hours each. Total hours are $5 \times 10=50$

[^1]:    TODOS has begun planning the TODOS 2018 conference.
    The leaders of the Planning Committee for this endeavor are Annette Kitagawa and Silvia Llamas-Flores. Would you like to be involved in planning this conference? If so, contact us at requests@todos-math.org.

